Rigid Body Dynamics

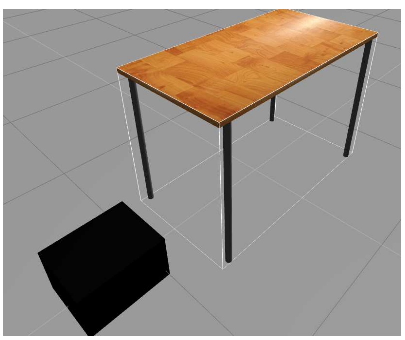
Tuesday, January 24, 2023 6:35 PM

Center of Mass:
The center of mass R is the centroid of a body with continuous mass and uniform density. It is also known as the center of gravity.

$$R = \frac{1}{M}\int_V \rho(r) r dV$$

Where:
M: the mass
ρ: the density

Experiment:
The box will be pushed into multiples steps until its center of mass goes beyond
the edge of the table ,so it's going to fall into the ground.



Kinetic energies:
The kinetic energy is due to the energy of the motion of a solid.
For a non-rotating object it can be determined as:

$$E_t = \frac{1}{2} M v_c^2$$

Where:
v_c: velocity of center of mass

If the body is rotating:

$$E_r = \frac{1}{2}\int_M v^2\, dm = \frac{1}{2}\int_M (r\omega)^2 dm = \frac{1}{2} I\omega^2$$

Where:
ω: angular velocity
r:distance of any mass dm from that line
I:body moment inertia

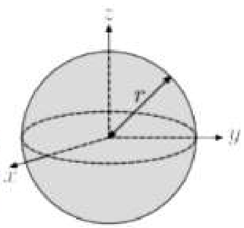
The total energy of a body's center mass is the sum of its translational and

rotation energy
$E_k = E_t + E_r$

Moments of Inertia:
Translational Inertia:
$F = ma$
Rotational Inertia:
$\tau = I\alpha$
Where:
τ: Torque
α: Angular acceleration

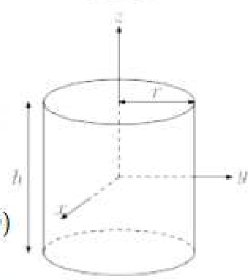
The moment of inertia is given by the integrating each differential mass element
$$I = \int_M r^2 dm$$
Where:
r: distance from axis of rotation
The same equation can be written in terms of density
$$I = \int_V r^2 p(r) dV$$

For the **same object**, **different axes of rotation will have different moments of inertia** about those axes. In general, the moments of inertia are not equal unless the object is symmetric about all axes. Let's see some examples extracted from Wikipedia's "List of moments of inertia" (density is assumed to be constant in all the cases).



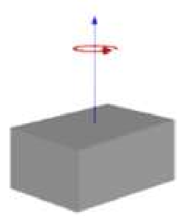
For a solid sphere or ball of radius r and mass m, the moment of inertia for any axis passing through the centroid is:

$$I = \frac{2}{5} mr^2 \qquad (9)$$



For a solid cylinder of radius r, height h and mass m, the moments of inertia about the main axes are:

$$I_z = \frac{1}{2} mr^2 \qquad (10)$$
$$I_x = I_y = \frac{1}{12} m(3r^2 + h^2) \qquad (11)$$

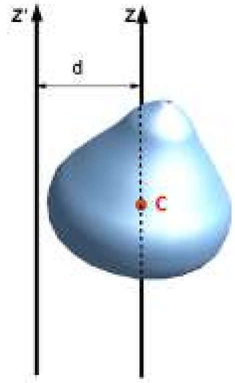


For a solid rectangular box of height h, width w, depth d, and mass m:

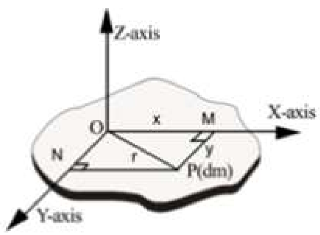
$$I_h = \frac{1}{12} m(w^2 + d^2) \qquad (12)$$
$$I_w = \frac{1}{12} m(d^2 + h^2) \qquad (13)$$
$$I_d = \frac{1}{12} m(w^2 + h^2) \qquad (14)$$

Parallel axis theorem
This theorem allow us to establish a new rotation axis, but it should be parallel to the center of

mas axis.
$I_{z'} = I_z + md^2$
Where:
I_z: Moment of inertia of center of mass
$I_{z'}$: Moment of inertia of new rotation axis



Parallel axis theorem
This theorem states that the moment of inertia of a perpendicular axis to the plane is the sum of the inertia moment of x and y.
$I_z = I_x + I_y$



Stretch rule (Routh's rule)
This rule states that if a rigid body is compressed or stretched to an axis rotation, its moment of inertia
won't change as long as the mass distribution doesn't change.

Experiment:
If a constant torque τ is applied for a fixed duration T, the angular acceleration α is constant during the inverval
of time and its value is:
$\alpha = \frac{\tau}{I}$
The final angular velocity ω will be:
$\omega = \omega_0 + \alpha T$
In this case the τ is 0.1 and T is 1

Moments of Inertia:

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0.0416667
0.056780260521
0.0683333
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